

Crossed Product Structure of Quantum Euclidean Groups

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Abstract

It is shown that quantum Euclidean groups $E_q(2)$, $E_\kappa(2)$ and $E_\kappa(3)$ have the structure of generalised crossed products.

1 Introduction

The idea of a crossed product, which appeared in studies of cohomology of algebras over a Hopf algebra [16] and the Hopf-Galois theory [1], may be summarised as follows. Given a Hopf algebra H and an algebra M one wants to construct an algebra P isomorphic to $M \otimes H$ as a vector space and such that P is a right H -comodule with a coaction $\Delta_R : P \rightarrow P \otimes H$, which is an algebra map trivial on M and coinciding with a coproduct on H . The complete answer to this problem is known [1]. Namely, one can construct such an algebra P if and only if there is a weak action $\rho : H \otimes M \rightarrow M$ of H on M and a 2-cocycle $\sigma : H \otimes H \rightarrow M$ which satisfy some conditions (see [1] for details).

Crossed-products appear in a variety of places in quantum group theory. Hopf-Galois extensions are understood as quantum group principal bundles [6] and a certain kind of a crossed product, known as a cleft extension, corresponds to a trivial quantum group principal bundle. The quantum double of Drinfeld is a special case of a crossed product, known as a double cross product [12] (it is also an example of a trivial quantum group principal bundle). Also, κ -Poincaré algebra [9] can be understood as a crossed product [13]. It seems reasonable to expect that inhomogeneous quantum groups which are built on a tensor product of two algebras should enjoy the structure of some kind of a crossed product. One quickly finds, however, that the standard definition of a crossed product is too restrictive for this purpose. Take $E_q(2)$ [17] as an example. As a vector space, $E_q(2)$ is built on a tensor product of the quantum hyperboloid X_q [15] and the Hopf algebra of formal power series in Z , $\mathbf{C}[Z, Z^{-1}]$. Furthermore $E_q(2)$ is a right $\mathbf{C}[Z, Z^{-1}]$ -comodule with a coaction that acts trivially on X_q and as a coproduct on $\mathbf{C}[Z, Z^{-1}]$. This coaction, however, **is not** an algebra map. Therefore, to interpret $E_q(2)$, and some other inhomogeneous groups as well, as a crossed product, one needs to generalise the notion of a crossed product in such a way that the condition that Δ_R be an algebra map gets relaxed. Since this is the only place where the algebra structure of H enters, to achieve a suitable generalisation of the crossed product we need to consider a crossed product by a **coalgebra** equipped with some additional structure, weaker than that of a Hopf algebra. This leads naturally to the concept of an entwining structure which was introduced in [7] in an attempt

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to construct a suitable quantum group gauge theory on quantum homogeneous spaces. In this context a generalisation of a crossed product was proposed in [4].

In this short note we summarise the construction of crossed products by a coalgebra of [4] and show that $E_q(2)$, $E_\kappa(2)$ and $E_\kappa(3)$ are such crossed products.

Throughout the paper we use standard Hopf algebra notations. In a coalgebra C , Δ denotes the coproduct and $\epsilon : C \rightarrow \mathbb{C}$ denotes the counit. We use the Sweedler notation to denote the coproduct in C , $\Delta c = c_{(1)} \otimes c_{(2)}$ (summation understood), for any $c \in C$. By convolution product we mean a product $*$ in a space of linear maps $C \rightarrow P$, where P is an algebra, given by $f * g(c) = f(c_{(1)})g(c_{(2)})$. A map $C \rightarrow P$ is said to be convolution invertible if it is invertible with respect to this product.

2 Crossed products by a coalgebra

We first recall the definition of an entwining structure from [7]. We say that a coalgebra C and an algebra P are *entwined* if there is a map $\psi : C \otimes P \rightarrow P \otimes C$ such that

$$\psi \circ (\text{id}_C \otimes \mu) = (\mu \otimes \text{id}_C) \circ \psi_{23} \circ \psi_{12}, \quad \psi(c \otimes 1) = 1 \otimes c, \quad \forall c \in C \quad (1)$$

$$(\text{id}_P \otimes \Delta) \circ \psi = \psi_{12} \circ \psi_{23} \circ (\Delta \otimes \text{id}_P), \quad (\text{id}_P \otimes \epsilon) \circ \psi = \epsilon \otimes \text{id}_P,$$

where μ denotes multiplication in P , and $\psi_{12} = \psi \otimes \text{id}_P$ and $\psi_{23} = \text{id}_P \otimes \psi$. We denote the action of ψ on $c \otimes u \in C \otimes P$ by $\psi(c \otimes u) = u_\alpha \otimes c^\alpha$ (summation understood).

Furthermore we assume that there is a group-like $e \in C$, i.e. $\Delta e = e \otimes e$, $\epsilon(e) = 1$ and a map $\psi^C : C \otimes C \rightarrow C \otimes C$ such that for any $c \in C$

$$(\text{id} \otimes \Delta) \circ \psi^C = \psi_{12}^C \circ \psi_{23}^C \circ (\Delta \otimes \text{id}), \quad (\text{id} \otimes \epsilon) \circ \psi^C = \epsilon \otimes \text{id}, \quad \psi^C(e \otimes c) = \Delta c,$$

where $\psi_{12}^C = \psi^C \otimes \text{id}_C$ and $\psi_{23}^C = \text{id}_C \otimes \psi^C$. We denote the action of ψ^C on $b \otimes c$ by $\psi^C(b \otimes c) = c_A \otimes b^A$ (summation understood).

With these assumptions P is a right C -comodule with a coaction $\Delta_R u = \psi(e \otimes u)$. Moreover the fixed point subspace $M = P_e^{coC} = \{x \in P \mid \Delta_R x = x \otimes e\}$ is a subalgebra of P . We call (P, C, ψ, e, ψ^C) the *entwining data*. A number of examples of entwining data may be found in [7]; all that are used in Section 3 come from the structure of quantum homogenous spaces [5].

Given the entwining data (P, C, ψ, e, ψ^C) and $M = P_e^{coC}$ the crossed product of M by C is defined as follows. Assume that there are linear maps $\sigma : C \otimes C \rightarrow M$ and $\rho : C \otimes P \rightarrow P$ such that for all $x, y \in M$, $a, b, c \in C$:

- (i) $\rho(e, x) = x$, $\rho(c, 1) = \epsilon(c)$;
- (ii) $\rho(c_{(1)}, (xy)_\alpha) \otimes c_{(2)}^\alpha = \rho(c_{(1)}, x_\alpha) \rho(c_{(2)}^\alpha_{(1)}, y_\beta) \otimes c_{(2)}^\alpha_{(2)}^\beta \in M \otimes C$;
- (iii) $\sigma(e, c) = \epsilon(c)$, $\sigma(c_{(1)}, e_A) \otimes c_{(2)}^A = 1 \otimes c$;
- (iv) $\rho(a_{(1)}, \sigma(b_{(1)}, c_A)_\alpha) \sigma(a_{(2)}^\alpha_{(1)}, b_{(2)}^A_B) \otimes a_{(2)}^\alpha_{(2)}^B = \sigma(a_{(1)}, b_A) \sigma(a_{(2)}^A_{(1)}, c_B) \otimes a_{(2)}^A_{(2)}^B$;
- (v) $\rho(a_{(1)}, \rho(b_{(1)}, x_\alpha)_\beta) \sigma(a_{(2)}^\beta_{(1)}, b_{(2)}^\alpha_A) \otimes a_{(2)}^\beta_{(2)}^A = \sigma(a_{(1)}, b_A) \rho(a_{(2)}^A_{(1)}, x_\alpha) \otimes a_{(2)}^A_{(2)}^\alpha$.

Then we can define an algebra structure on $M \otimes C$ with product

$$(x \otimes b)(y \otimes c) = x \rho(b_{(1)}, y_\alpha) \sigma(b_{(2)}^\alpha_{(1)}, c_A) \otimes b_{(2)}^\alpha_{(2)}^A.$$

and unit $1 \otimes e$. This algebra is denoted by $M \rtimes_{\rho, \sigma} C$ and called a *crossed product* by a coalgebra C . The pair (ρ, σ) is called the *crossed product data* for the entwining data (P, C, ψ, e, ψ^C) .

Therefore, to construct a crossed product $M \rtimes_{\rho, \sigma} C$ one first needs to introduce an entwining structure and then maps ρ, σ that satisfy conditions (i-v). One can easily convince oneself that all the previously known crossed products fit into above scheme. For example, if one takes $C = H$ to be a Hopf algebra, $e = 1$, and P a right H -comodule algebra with a coaction $\Delta_R : P \rightarrow P \otimes H$ then one can define a natural entwining structure by $\psi(h \otimes u) = u^{(0)} \otimes hu^{(1)}$, $\psi^C(h \otimes g) = g_{(1)} \otimes hg_{(2)}$, where $\Delta_R u = u^{(0)} \otimes u^{(1)}$. With this entwining structure the conditions listed above become the standard conditions for a weak action ρ and a cocycle σ [1].

Similarly, if $C = B$ is a braided group, P a right braided B -comodule algebra with braiding Ψ , then we take $e = 1$ and define the entwining structure by $\psi(b \otimes u) = \Psi(b \otimes u^{(0)})u^{(1)}$, $\psi^C(b \otimes c) = \Psi(b \otimes c_{(1)})c_{(2)}$. In this case the notion of a crossed product by B introduced above, is equivalent to the notion of a crossed product by a braided group [11].

A truly new crossed product is constructed assuming that there is a convolution invertible map $\Phi : C \rightarrow P$ such that, for given entwining data (P, C, ψ, e, ψ^C) , $\Phi(e) = 1$ and

$$\psi \circ (\text{id}_C \otimes \Phi) = (\Phi \otimes \text{id}_C) \circ \psi^C. \quad (2)$$

In this case one can define the crossed product data (ρ, σ) by

$$\rho(c, u) = \Phi(c_{(1)})u_\alpha \Phi^{-1}(c_{(2)}^\alpha), \quad \sigma(b, c) = \Phi(b_{(1)})\Phi(c_A)\Phi^{-1}(b_{(2)}^A). \quad (3)$$

This crossed product is isomorphic to P as an algebra. From the point of view of a coalgebra gauge theory $P = M \rtimes_{\rho, \sigma} C$ is a trivial coalgebra bundle on M [7], while from the point of view of coalgebra extensions it is a *cleft extension* of M . In the next section it will be shown that $E_q(2)$, $E_\kappa(2)$ and $E_\kappa(3)$ are crossed products of this type.

Given the entwining data (P, C, ψ, e, ψ^C) , crossed product data (ρ, σ) , and a convolution invertible map $\gamma : C \rightarrow M$, such that $\gamma(e) = 1$ and

$$\psi_{23}^C \circ \psi_{12} \circ (\text{id}_C \otimes \gamma \otimes \text{id}_C) \circ (\text{id}_C \otimes \Delta) = (\gamma \otimes \text{id}_C \otimes \text{id}_C) \circ (\Delta \otimes \text{id}_C) \circ \psi^C,$$

one can construct a new crossed product with data (ρ', σ') ,

$$\begin{aligned} \rho'(c, u) &= \gamma(c_{(1)})\rho(c_{(2)}, u_\alpha)\gamma^{-1}(c_{(3)}^\alpha), \\ \sigma'(b, c) &= \gamma(b_{(1)})\rho(b_{(2)}, \gamma(c_{A(1)})_\alpha)\sigma(b_{(3)}^\alpha, c_{A(2)})\gamma^{-1}(b_{(4)}^A). \end{aligned}$$

The crossed product data (ρ, σ) and (ρ', σ') lead to the algebras which are isomorphic to each other and therefore (ρ, σ) and (ρ', σ') are said to be *gauge equivalent*. From the point of view of the coalgebra gauge theory, the map γ is a gauge transformation of a trivial principal bundle. The equivalence classes of (ρ, σ) define a ‘‘cohomology’’ which generalises the non-Abelian cohomology introduced in [10].

3 Crossed product structure of quantum Euclidean groups

3.1 The q -Euclidean group $E_q(2)$

This example is discussed in detail in [4]. The entwining data are as follows. The coalgebra C is spanned by group-like c_p , $p \in \mathbf{Z}$. It can be therefore equipped with an algebra structure of

$\mathbf{C}[Z, Z^{-1}]$ with $Z^p = c_{p+s}$ for any fixed $s \in \mathbf{Z}$. The total algebra P is taken to be $E_q(2)$ and is generated by v, v^{-1}, n_+, n_- subject to the following relations

$$vn_{\pm} = q^2 n_{\pm} v, \quad n_+ n_- = q^2 n_- n_+, \quad vv^{-1} = v^{-1}v = 1. \quad (4)$$

As is well-known $E_q(2)$ is a quantum group, and thus it has a Hopf algebra structure. For our purposes however, the Hopf algebra structure of $E_q(2)$ is not important. We need the entwining structure on $E_q(2)$ and C and this is provided by the map $\psi : C \otimes P \rightarrow P \otimes C$ given by

$$\psi(c_p \otimes v^{\pm 1}) = v^{\pm 1} \otimes c_{p\pm 1}, \quad \psi(c_p \otimes n_{\pm}) = n_{\pm} \otimes c_p + \mu_{\pm} q^{2p} v^{\pm 1} \otimes c_p - \mu_{\pm} q^{2p} v^{\pm 1} \otimes c_{p\pm 1}$$

where μ_+, μ_- are non-zero complex numbers, and extended to the whole of $E_q(2)$ by (1). It is an easy exercise to verify that such an extension is compatible with (4). Furthermore we take $e = c_s$, and $\psi^C(c_p \otimes c_r) = c_r \otimes c_{p+r-s}$.

Given this entwining structure we can define a coaction Δ_R of C on $E_q(2)$, by $\Delta_R(u) = \psi(c_s \otimes u)$. Following [3] one easily finds that the fixed point subalgebra M of P is generated by $z_{\pm} = v^{\pm 1} + \mu_{\pm}^{-1} q^{-2s} n_{\pm}$, which satisfy the relation $z_+ z_- = q^2 z_- z_+ + (1 - q^2)$. Therefore M is isomorphic to the quantum hyperboloid X_q [15].

To reveal the crossed product structure of $E_q(2)$ we define a linear map $\Phi : C \rightarrow E_q(2)$ by $\Phi(c_p) = v^{p-s}$. This map is clearly convolution invertible and it satisfies (2). Therefore $E_q(2) = X_q \rtimes_{\rho, \sigma} C$ with a trivial cocycle $\sigma(c_p, c_r) = 1$ and the map $\rho : C \otimes E_q(2) \rightarrow E_q(2)$,

$$\rho(c_p, v^{\pm 1}) = 1, \quad \rho(c_p, n_{\pm}) = q^{2p} (q^{-2s} n_{\pm} + \mu_{\pm} (v^{\pm 1} - 1)).$$

One can easily prove that, even if one introduces an algebra structure on C compatible with Δ , $E_q(2)$ is never a C -comodule algebra. Furthermore, because $\psi(C \otimes X_q)$ is *not* a subset of $X_q \otimes C$ there is no braiding making $E_q(2)$ a braided C -comodule algebra. Therefore the notion of a coalgebra crossed product developed in [4] and summarised in this paper is truly needed for description of internal structure of $E_q(2)$.

3.2 The κ -Euclidean group $E_{\kappa}(2)$

Similarly as for the $E_q(2)$ case, the coalgebra C is spanned by group-like elements c_p , $p \in \mathbf{Z}$. The κ -deformation $E_{\kappa}(2)$ of two-dimensional Euclidean group, which is obtained by the contraction of $SU_q(2)$ [8] or, equivalently, by quantisation of the Poisson structure on $E(2)$ [14], is generated by w, w^{-1}, a_1, a_2 , and its algebra structure is determined by the relations

$$[w, a_1] = \frac{\kappa}{2} (w - 1)^2, \quad [w, a_2] = i \frac{\kappa}{2} (w^2 - 1), \quad [a_1, a_2] = i \kappa a_1.$$

It is clear that a_1, a_2 generate a subalgebra of $E_{\kappa}(2)$. This subalgebra, known as a κ -plane, is a homogeneous space of $E_{\kappa}(2)$ [2]. We will denote it by \mathbf{R}_{κ}^2 .

We choose $e = c_0$ and proceed to define the entwining maps $\psi : C \otimes E_{\kappa}(2) \rightarrow E_{\kappa}(2) \otimes C$ and $\psi^C : C \otimes C \rightarrow C \otimes C$. The former is given by

$$\psi(c_p \otimes w^{\pm 1}) = w^{\pm 1} \otimes c_{p\pm 1}, \quad \psi(c_p \otimes a_{\pm}) = a_{\pm} \otimes c_p + \kappa p w^{\mp 1} \otimes (c_{p\mp 1} - c_p),$$

where $a_{\pm} = a_1 \pm i a_2$, and extended to the whole of $E_{\kappa}(2)$ by (1). One way of seeing that the map ψ is well-defined is to use the standard coproduct of $E_{\kappa}(2)$ [2] and observe that for

any $u \in E_\kappa(2)$, $\psi(c_l \otimes u) = u_{(1)} \otimes \pi(w^l u_{(2)})$, where $\pi : E_\kappa(2) \rightarrow C$ is a right module surjection given by $\pi(w^l) = c_l$, $\pi(a_1) = \pi(a_2) = 0$. By [7, Example 2.5] such a map necessarily entwines $E_\kappa(2)$ with C . This can also be verified directly. The map ψ^C is given by $\psi^C(c_p \otimes c_r) = c_r \otimes c_{p+r}$.

Using the map ψ we define a right C -comodule structure on $E_\kappa(2)$, $\Delta_R(u) = \psi(c_0 \otimes u)$. The fixed point subalgebra M of $E_\kappa(2)$ under this coaction is generated by a_1, a_2 and therefore it is isomorphic to the κ -plane \mathbf{R}_κ^2 .

Finally, the crossed product structure of $E_\kappa(2)$ is provided by the map $\Phi : C \rightarrow E_\kappa(2)$ given by $\Phi(c_p) = v^p$. This map is clearly convolution invertible, $\Phi(c_0) = 1$ and the condition (2) is satisfied. Therefore we have the crossed product $E_\kappa(2) = \mathbf{R}_\kappa^2 \rtimes_{\rho, \sigma} C$ with a trivial cocycle $\sigma(c_p \otimes c_r) = 1$ and the map $\rho : C \otimes E_\kappa(2) \rightarrow E_\kappa(2)$, given explicitly by

$$\rho(c_p \otimes w^{\pm 1}) = 1, \quad \rho(c_p \otimes a_i) = a_i.$$

Also in this case it is impossible to interpret $E_\kappa(2)$ as a C -comodule (braided) algebra, and thus $E_\kappa(2)$ is truly an example of a crossed product by a coalgebra.

3.3 The κ -Euclidean group $E_\kappa(3)$

Finally we sketch the crossed product structure of the deformation of three-dimensional Euclidean group. $E_\kappa(3)$ is obtained by a contraction of $SO_q(4)$, and is generated by the Euler angles α, β, γ , and ‘coordinates’ x_1, x_2, x_3 which satisfy the following relations [8]:

$$[\beta, x_+] = \kappa \sin \beta \tan(\beta/2), \quad [\beta, x_3] = \kappa \sin \beta, \quad [x_-, \omega] = 2\kappa \tan(\beta/2), \quad (5)$$

$$[x_3, x_+] = \kappa x_3, \quad [x_-, x_+] = \kappa x_-, \quad (6)$$

where $x_+ = x_1 \cos \alpha + x_2 \sin \alpha$, $x_- = -x_1 \sin \alpha + x_2 \cos \alpha$, $\omega = \alpha + \gamma$ and all other commutators vanish. The generators x_\pm, z span subalgebra of $E_\kappa(3)$ which is denoted by \mathbf{R}_κ^3 . $E_\kappa(3)$ is a Hopf algebra and \mathbf{R}_κ^3 is its homogeneous space, thus we use [7, Example 2.5] to construct the entwining structure on $E_\kappa(3)$. Define a right ideal $J \subset E_\kappa(3)$ generated by x_\pm, z and consider $C = E_\kappa(3)/J$. C is a coalgebra which can be equipped with a Hopf algebra structure of functions on $SO(3)$ and is generated by the rotation matrix A expressed in terms of the Euler angles α, β, γ . The entwining structure is defined by $\psi(c \otimes u) = u_{(1)} \otimes \pi(j(c)u_{(2)})$, where π is a natural surjection $E_\kappa(3) \rightarrow C$ and j is a (Hopf algebra) inclusion $C \hookrightarrow E_\kappa(3)$. Denoting elements of A by a_{ij} , and using the fact that both $E_\kappa(3)$ and C are matrix quantum groups, we thus find

$$\psi(a_{ij} \otimes x_k) = x_k \otimes a_{ij} + \sum_n a_{kn} \otimes [a_{ij}, x_n], \quad \psi(a_{ij} \otimes a_{kl}) = \sum_n a_{kn} \otimes a_{ij} a_{nl}, \quad (7)$$

where the $SO(3)$ subgroup of $E_\kappa(3)$ is identified with C via π and j . Equation (7) can be made more explicit by representing a_{ij} in terms of the Euler angles and using (5-6). Furthermore we take $\psi^C(b \otimes c) = c_{(1)} \otimes bc_{(2)}$ and $e = 1$. Since π is not an algebra map, the coaction $\Delta_R(u) = \psi(1 \otimes u)$ is not an algebra map either, and to interpret $E_\kappa(3)$ as a crossed product built on $\mathbf{R}_\kappa^3 \otimes C$, one truly needs the construction described in Section 2. This is provided by

the map $\Phi \equiv j$, which clearly satisfies all the required conditions and thus makes $E_\kappa(3)$ into a crossed product $\mathbf{R}_\kappa^3 \rtimes_{\rho, \sigma} C$ with a trivial cocycle $\sigma(b, c) = \epsilon(b)\epsilon(c)$ and the 'action'

$$\rho(c, x_i) = \epsilon(c)x_i, \quad \rho(b, c) = \epsilon(b)\epsilon(c),$$

for any $b, c \in C$.

4 Conclusions

In this short note we described crossed product structure of three examples of inhomogeneous quantum groups. It is clear that other inhomogeneous quantum groups may enjoy the similar structure, too (for example, formula (7) suggests an obvious generalisation). We think that analysis of inhomogeneous quantum groups from the point of view of their crossed product structure may enhance our understanding of these groups, and thus indicate new applications to physics, special functions and algebra. It can also lead to development of gauge theories on such quantum groups along the lines of [7].

References

- [1] R.J. Blattner, M. Cohen and S. Montgomery, *Trans. A.M.S.* **298** (1989) 671–711. R.J. Blattner and S. Montgomery, *Pacific J. Math.* **137** (1986) 37–54. Y. Doi and M. Takeuchi, *Commun. Alg.* **14** (1986) 801–817. Y. Doi, *Commun. Alg.* **17** (1989) 3053–3085.
- [2] F. Bonechi *et.al.*, “A new class of deformed special functions from quantum homogeneous spaces”, Preprint q-alg/9605027.
- [3] F. Bonechi *et.al.*, *Commun. Math. Phys.* **175** (1996) 161–176.
- [4] T. Brzeziński, “Crossed products by a coalgebra”, Preprint q-alg/9603016.
- [5] T. Brzeziński, *J. Math. Phys.* **37** (1996) 2388–2399.
- [6] T. Brzeziński and S. Majid, *Commun. Math. Phys.* **157** (1993) 591–638. Erratum **167** (1995) 235. M. Pflaum, *Commun. Math. Phys.* **166** (1994) 279–315.
- [7] T. Brzeziński and S. Majid, “Coalgebra gauge theory”, Preprint q-alg/9602022.
- [8] E. Celeghini *et. al.*, *J. Math. Phys.* **31** (1990) 2548–2551. *J. Math. Phys.* **32** (1991) 1159–1165.
- [9] J. Lukierski *et. al.*, *Phys. Lett. B* **268** (1991) 331–338. *Phys. Lett. B* **293** (1992) 344–352.
- [10] S. Majid, in: *Quantum Groups, Integrable Statistical Models and Knot Theory*, eds. M-L. Ge and H.J. de Vega, p. 231–282, (World Scientific, 1993).
- [11] S. Majid, *J. Algebra* **163** (1994) 165–190.
- [12] S. Majid, *J. Algebra* **130** (1990) 17–64.
- [13] S. Majid and H. Ruegg, *Phys. Lett. B* **334** (1994) 348–354.
- [14] P. Maślanka, *J. Math. Phys.* **35** (1994) 1976–1983.
- [15] K. Schmüdgen, *Commun. Math. Phys.* **159** (1994) 217–238.
- [16] M. Sweedler, *Trans. AMS* **133** (1968) 205–239.
- [17] L.L. Vaksman and L.I. Korogodskii, *Soviet Math. Dokl.* **39** (1990) 173–178. S.L. Woronowicz, *Commun. Math. Phys.* **136** (1991) 399–432.